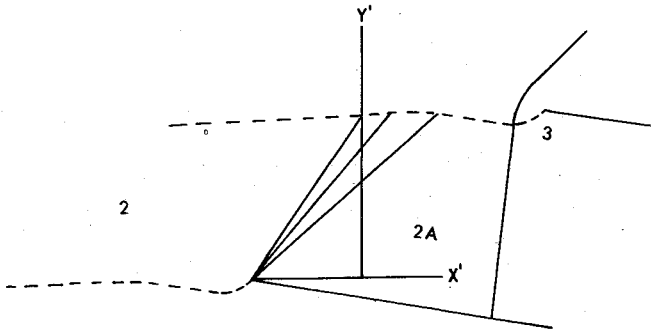

Fig. 2 Bow wave, passage shock, and Prandtl-Meyer expansion.

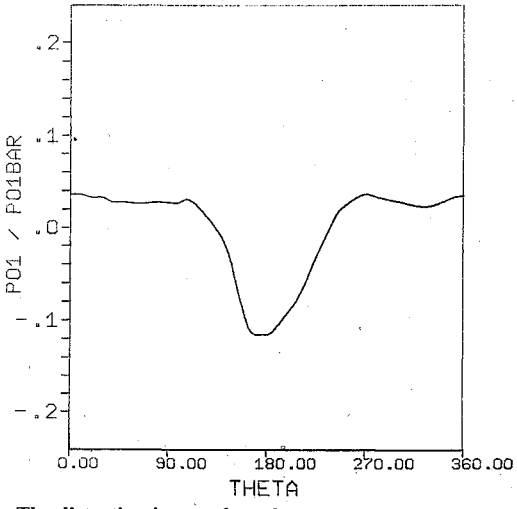

Fig. 3 The distortion imposed on the compressor stage of (2), 100% design speed, maximum mass flow.

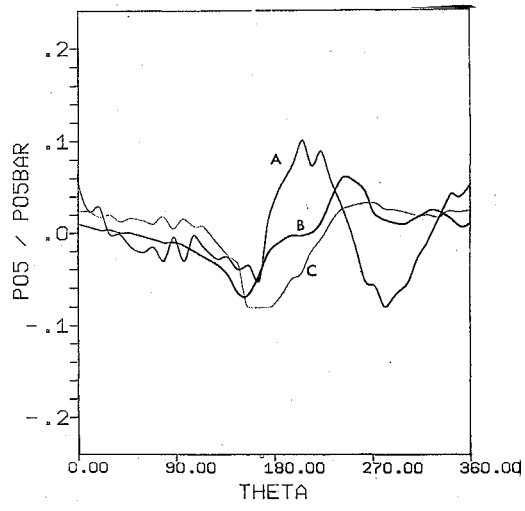

Fig. 4 Stagnation pressure perturbation at station 5, theory and experiment: curve A is theory taken with finite chord, $\xi_{\text{eff}} = \xi_R - (s\sin\theta_I - \bar{L})$; curve B in the experiment and curve C is theory taken with zero chord.

relative Mach numbers, there is, at least for the sample case calculated, a strong amplificaiton of stagnation pressure perturbations with increasing chord/mean radius (increasing reduced frequency). It is perhaps this amplification which is showing up in the higher harmonics evident in Fig. 4. Figure 5 shows the stagnation temperature perturbations at station 5 (between rotor and stator).

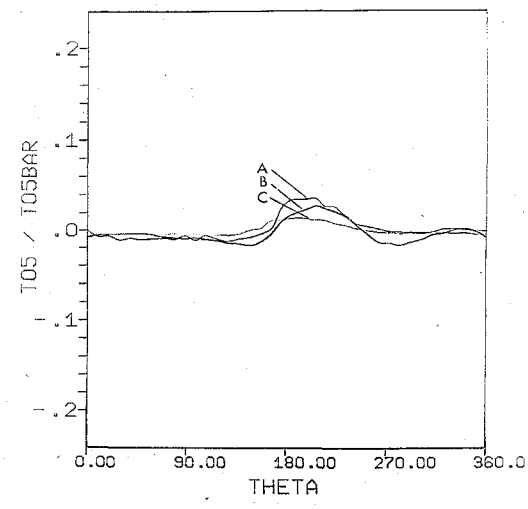

Fig. 5 Stagnation temperature perturbation at station 5, theory and experiment: curve A is theory taken with finite chord, $\xi_{\text{eff}} = \xi_R - (s\sin\theta_I - \bar{L})$; curve B is the experiment; and curve C is theory taken with zero chord.

Acknowledgments

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Linearized Unsteady Flame Surface Approximation Result in Complex Notation

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Nomenclature

$f_i(r)$ = complex functions of position vector r, $i = \alpha_f, \alpha_{ox}, \beta$

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r = position vector
t = time
Y_{ox}, Y_f = mass fractions of oxidizer and fuel, respectively
α_{ox}, α_f = variables directly related to Y_{ox}, and Y_f, respectively, by means of some constants
α_t = complex constant
β = $\alpha_{ox} - \alpha_f$
$\bar{(\ )}$ = steady-state component
$(\ )'$ = unsteady perturbation
Im = imaginary part
Re = real part

Subscripts

f = fuel
ox = oxidizer

Introduction

THE steady-state flame surface approximation result [Ref. 1, Chap. 3, Eqs. (23) and (24)] is a useful device which permits one to obtain species concentration and temperature profiles in problems involving diffusion flames. The objective of this Note is to derive its analog in the linearized unsteady case in complex notation. Such a derivation will be useful in treating unsteady problems involving diffusion flames. The derivation is both mathematically and physically consistent with the linearization procedure, and is not found in any of the available literature.

Derivation

The problem under consideration is illustrated in Fig. 1. The unsteady components of α_{ox}, α_f, and β are taken to be of the general form:

$$f_i(r)e^{\alpha_t t} \qquad (i=\alpha_{ox},\alpha_f,\beta)$$

The flame surface approximation simply states that on one side of the flame only fuel is present and on the other side of the flame only oxidizer is present.

A precise translation of the preceding statement into mathematical form results in

$$\begin{aligned} \mathrm{Re}(\alpha_f) &= -\mathrm{Re}(\beta) && \text{if } \mathrm{Re}(\beta)>0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{1}$$

$\mathrm{Re}(\beta)=0$ defines the instantaneous position of the flame surface at any given time t. Consequently, a zone of flame oscillation can be defined as the region at every point of which $\mathrm{Re}(\beta)$ passes through the value zero at some time.

In deriving a linearized version of Eq. (1) it is necessary to note that in many unsteady problems it becomes necessary to apply one or more conditions at the instantaneous position of an oscillating surface. In linear unsteady analysis the usual procedure is to apply the same conditions at the steady-state position of the oscillating surface. An example from acoustics is the problem of the oscillating piston in an infinite baffle. A similar approach will be adopted here. Consequently, the equations derived here will be exact outside of the zone of flame oscillation but will be inaccurate inside this zone. This cannot be avoided in a linear theory and the effects of these inaccuracies are of higher order in the perturbed quantities.

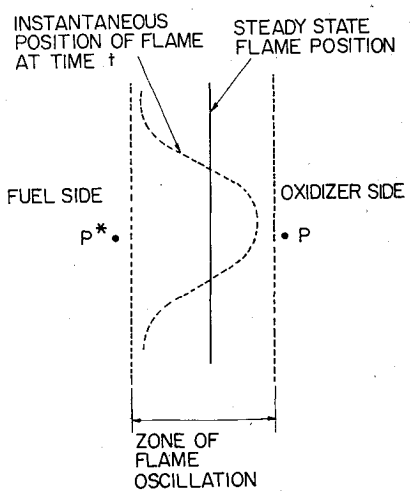

Fig. 1 Neighborhood of the zone of flame oscillation.

It follows from the definition of the zone of flame oscillation that

$$|\bar{\beta}| > |\mathrm{Re}(\beta')| \tag{2}$$

outside of the zone of flame oscillation.

It follows from Eq. (2) that outside of the zone of flame oscillation:

$$\bar{\beta}>0 \Rightarrow \mathrm{Re}(\beta)>0$$

and

$$\bar{\beta}<0 \Rightarrow \mathrm{Re}(\beta)<0$$

and vice versa.

Thus outside of the zone of flame oscillation $\mathrm{Re}(\beta)$ and $\bar{\beta}$ have the same sign. Consequently a simplification of Eq. (1), which is exact outside of the zone of flame oscillation, is

$$\begin{aligned} \mathrm{Re}(\alpha_f) &= -\mathrm{Re}(\beta) && \text{if } \bar{\beta}>0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{3}$$

Using the steady-state flame surface approximation result

$$\begin{aligned} \bar{\alpha}_f &= -\bar{\beta} && \text{if } \bar{\beta}>0 \\ &= 0 && \text{otherwise} \end{aligned}$$

Equation (3) can be simplified further to give

$$\begin{aligned} \mathrm{Re}(\alpha_f') &= -\mathrm{Re}(\beta') && \text{if } \bar{\beta}>0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{4}$$

Once again Eq. (4) is exact outside of the zone of flame oscillation. Any point P is now considered outside the zone of flame oscillation on the oxidizer side of the flame. Application of Eq. (4) at point P leads to the conclusion that

$$\mathrm{Re}(\alpha_f')=0 \quad \text{at point P at all times}$$

Unless the frequency of the oscillation is zero this implies that

$$\mathrm{Im}(\alpha_f')=0 \quad \text{at point P at all times}$$

Hence Eq. (4) becomes

$$\begin{aligned} \mathrm{Re}(\alpha_f') &= -\mathrm{Re}(\beta') && \text{if } \bar{\beta}>0 \\ \alpha_f' &= 0 && \text{otherwise} \end{aligned} \tag{5}$$

Any point P* is now considered outside the zone of flame oscillation on the fuel side of the flame. Application of Eq. (5) [or Eq. (4)] at point P* leads to the conclusion that

$$\mathrm{Re}(\alpha_f'+\beta')=0 \qquad \text{at point P* at all times}$$

Unless the frequency of the oscillation is zero this implies that

$$\mathrm{Im}(\alpha_f'+\beta')=0 \qquad \text{at point P* at all times}$$

that is,

$$\mathrm{Im}(\alpha_f') = -\mathrm{Im}(\beta') \qquad \text{at point P* at all times}$$

Hence Eq. (5) becomes

$$\alpha_f' = -\beta' \quad \text{if} \quad \bar{\beta} > 0$$
$$= 0 \quad \text{otherwise} \tag{6}$$

Equation (6) is the linearized unsteady flame surface approximation result in complex notation and is exact outside the zone of flame oscillation.

Acknowledgment

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Buckling of Thin Plates Using the Collocation Least-Square Method

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Introduction

THE buckling behavior of thin plate elements forming webs and flanges of spars, ribs, and other portions of flight vehicles are of prime importance in the aerospace industry. Traditionally, the stability of thin plates has been analyzed by the finite difference and Rayleigh-Ritz methods.[1,2] In this Note, a relatively new numerical approach utilizing the collocation least-square concept is used to obtain a solution for buckling of clamped rectangular plates subjected to in-plane normal loads. The success of the collocation least-square method used in the present Note has already been demonstrated previously in applications to various linear and nonlinear boundary-value problems in plate bending.[3,4]

Analysis

The conventional method of collocation was first discussed in a report by Frazer et al.[5] To illustrate the method, consider a linear eigenvalue problem defined by the differential equation

$$L(W) - \lambda M(W) = 0 \tag{1}$$

where L and M are differential operators, and λ the eigenvalue. The approximate solution to Eq. (1) can be expressed as

$$W = \sum_{j=1}^{n} a_j u_j(x,y) \tag{2}$$

where a_j are undetermined coefficients and u_j are independent functions.

In conventional interior collocation, W would satisfy the prescribed boundary conditions but not the governing differential equation. The parameters $a_1,\ldots,a_n$ and the eigenvalue are evaluated by requiring the residual function, resulting from the substitution of W into Eq. (1), to be zero at n arbitrarily chosen collocation points inside the region defined by the problem. As pointed out by Collatz,[6] the choice of location of collocation points required by such a procedure is a matter of some uncertainty and the results obtained can fluctuate greatly for arbitrary choices of collocation points. This fluctuation of results can be reduced by the application of the collocation least-square concept. In applying this method, the residual function is set up at m collocation points, where $m \gg n$. This results in a system of m simultaneous equations expressed in matrix notation as:

$$[L - \lambda M]\{a\} = \{r\} \tag{3}$$

or

$$[C]\{a\} = \{r\} \tag{4}$$

where $[L]$ and $[M]$ are $m \times n$ matrices, $\{a\}$ the $n \times 1$ vector of unknown coefficients, and $\{r\}$ the $m \times 1$ vector of associated errors at m collocation points,

A least-square solution to the resulting set of equations would be that which yields a minimum value of E.

$$E = \sum_{i=1}^{m} r_i^2 = \{r\}^T\{r\} = \{a\}^T[C^TC]\{a\} \tag{5}$$

There are two alternative least-square procedures to minimize the error function E. E can be differentiated with respect to the n parameters, $(a_1, a_2,\ldots,a_n)$ resulting in $[C^TC]\{a\} = 0$, and λ is determined by setting the determinant of $[C^TC]$ equal to zero. Since the elements of $[C^TC]$ depend on λ and λ^2, solutions of the characteristic equation may take on complex values even when the actual eigenvalues for the problem are real. However, when the eigenvalues are known to be real, the real part of the complex results may furnish a good approximation.

The alternative least-sqaure scheme begins with imposing the normalization condition, which corresponds to choosing the value of one parameter, e.g., $a_1 = 1$. Then the error function E is differentiated with respect to a different set of parameters.

$$(\{a\}^j, \lambda^j) = (a_2^j, a_3^j, \ldots, a_n^j, \lambda^j)$$

where λ^j represents the jth eigenvalues associated with the jth eigenfunction, which is approximated by $W = (x,y,1,a_2^j,a_3^j,\ldots,a_n^j)$. In this approach, the error function E is treated as a nonlinear function of the eigenvalue λ, having multiple local minima, each of which corresponds to a distinct eigenvalue λ^j.

Rewriting

$$E = \{a\}^T[L - \lambda M]^T[L - M]\{a\} = \{a\}^T[P - 2\lambda Q + \lambda^2 S]\{a\} \tag{6}$$

where $P = [L]^R[L]$, $Q = \frac{1}{2}[L^TM + M^TL]$, and $[S] = [M]^T[M]$, and setting partial derivatives of E with respect to λ and $\{a\}$ equal to zero leads to the coupled equations:

$$\lambda = \{a\}^T[Q]\{a\} / \{a\}^T[S]\{a\} \tag{7}$$

$$[P - 2\lambda Q + \lambda^2 S]\{a\} = 0 \tag{8}$$

The iteration scheme starts with an approximate value of λ to calculate $\{a\}$ from Eq. (8), then Eq. (7) is used to find an improved value of λ, and the process continues to results of desired accuracy.

While the least-square augmented collocation method formulated in this Note can be applied to a wide variety of structural buckling problems, for the sake of brevity, only the solution of a clamped rectangular plate is illustrated.

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